# Laminar Fully Developed Forced Convection of Power Law Fluid in a Circular Tube

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## Abstract

Laminar forced convection heat transfer in a non-Newtonian fluid flow inside a pipe has been investigated analytically. Fully developed laminar velocity distributions obtained by a power-law fluid rheology model is used, and viscous dissipation was taken into account. The theoretical analysis of the heat transfer is performed under a constant wall temp. case. An important feature of this approach is that it permits an arbitrary distribution of the surrounding medium temperature and an arbitrary velocity distribution of the fluid. These techniques were verified by a comparison with the existing results. The effects of the Brinkman number and rheological properties on the distribution of the local Nusselt number have been studied. It is shown that the Nusselt number strongly depends on the value of power law index. The Nusselt number sharply decreases in the range of $0 < n < 0.1$. However, for $n > 0.5$, the Nusselt number decreases monotonically with the increasing n, and for $n > 1$, the values of Nusselt number approach a constant value.

**Keywords:** Heat transfer, forced convection, Non-Newtonian fluid, analytical solution, viscous dissipation, Graetz's problem, laminar flow.

**المستخلص**

تم في هذا البحث دراسة نظرية لعملية انتقال الحرارة بالحمل القسري لجريان مائع غير نيوتوني متشكل هايدرودينيميكيا داخل انبوب دائري . المائع تم دراسته في مرحلة التشكيل التام هايدرودينميكيا واخذت فرضية الرفع للقوة كموديل لدراسة المائع .الدراسة النظرية اخذت فرضية ثبوت درجة الحرارة على الجدار وتمت دراسة مخططات السرعة ودرجات الحراة لتوزيعات مختلفة داخل الانبوب الدائريومعرفة تاثير ذلك على رقم نيسيلت .اعتمد الحل النظري على درجة الحرارة على الجدار .واستخدمت عده فرضيات اخرى للحل وقورنت بالنتائج .تم دراسية تاثير رقم برونكمان على رقم نيسلت ,ووجد ان رقم نيسلت يعتمد على قانون الرفع للأس ويقل عندما تكون قيمة اللأس اكبر من صفر واقل من العشر لذلك رقم نيسلت للاس اكبرمن خمسة اعشار يقل مع زياده قيمه اللاس و للقيم اكبر من الواحد .

## 1. Introduction

An understanding of convection heat transfer in non-Newtonian fluids inside pipes is crucial to the design of several types of thermal equipments. From this viewpoint, heat transfer problems of this type have been investigated by a number of researchers [1,2,3]. The problem pertaining to the derivation of the local Nusselt number in the thermal region when an incompressible fluid flows through a pipe with a fully developed velocity distribution is of particular interest; this problem is referred to as the Graetz problem. It has attracted the interest of not only engineers but also applied mathematicians because of the difficulties involved in deriving its solution. The original Graetz problem, which was first analytically solved by Graetz [4], is refer to the classical Graetz –Nusselt problem in single phase flow that neglects the effects of axial heat conduction, viscous dissipation, and thermal energy sources within the fluid. It is regarded as one of the most important solutions in the heat transfer science and it governs forced convection heat transfer for fluid with known velocity profile and involves finding of the heat transfer rate in a fully developed flow of fluid flowing inside conduit of various cross-sectional geometries with constant heat flux or constant wall temperature mode of heating. This type of solution allows temperature profile to be calculated from the coupled equations of motion andenergy [5].

A comprehensive analytically studies for the fully developed power-law fluid flowing in a circular tube for both uniform wall heat flux and wall temperature has been done [6] , but the authors neglected the effects of viscous dissipation. They show that the value of Nusselt number for a power-law fluid within uniform heat flux is given by:

$$Nu = \frac{8(3n+1)(5n+1)}{31n^2+12n+1} \quad (1)$$

Where $n$ is the power-law index. For Newtonian fluid, i.e. for $n=1$, Eqn. (1) yields the well known result as:

$$Nu = \frac{48}{11} = const. \quad (2)$$

Where this result in Eqn. (2) was confirmed early by many number of authors [7,8].

However, the Graetz problem has been extended over problems that focus on turbulent flows, non-Newtonian flows, forced convection in a porous medium, and the effects of viscous dissipation for Newtonian fluid and that include effects of heat conduction [9-17]. In all the works cited above there is no studied related to the effects of viscous dissipation on heat transfer with non-Newtonian fluid.

Therefore, the objective of this study is to mathematically solve the forced convection heat transfer problem in a pipe subjected to constant wall temp. for fully developed region, which is a type of Graetz problem, and derive completely analytical solutions for the fluid temperature profile and local Nusselt number. Since the present study focuses on heat transfer with a sufficiently large Peclet number $(Pe)$, the axial heat conduction is considered negligible. However, viscous dissipation is taken into account. Numerical calculations are performed to demonstrate the effects of the Brinkman number $(Br)$ and rheological properties on the distribution of the local Nusselt number

## 2. Mathematical Model and Formulation

Figure (1)shows the physical model and coordinate system.A non-Newtonian fluid with fully developed velocity profile $u(r)$ flows into a cylindrical pipe of radius $R$ .The pipe is convectively heated or cooled by the surrounding medium of constant wall temp. $q_w$.

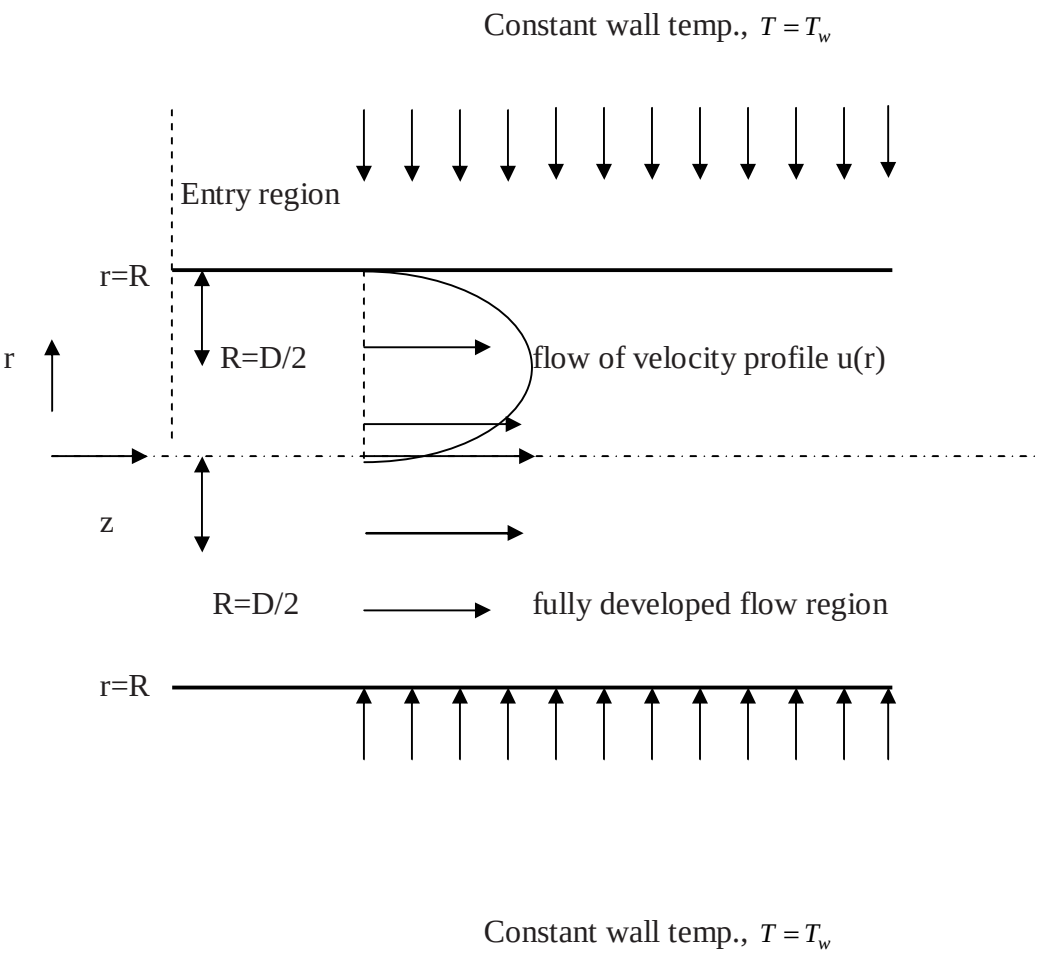


**Figure (1). Notations and axes of the problem.**

According to the type of fluid flowing inside the pipe, a power-law fluid, which can approximate the non-Newtonian viscosity of many types of fluids with good accuracy over a wide range of shear rates, is considered here. The shear stress $(t_{rz})$ acting on the viscous fluid is given by the formula [18]:

$$t_{rz} = -m\left|\frac{du}{dr}\right|^{n-1}\frac{du}{dr} \tag{3}$$

Where $m$ and $n$ are the power-law model parameter index, respectively. Depending on power law index $(n)$; there are three cases as:

1. $n<1$ indicates that the fluid is a pseudo-plastic fluid,

2. $n=1$ indicates that the fluid is equivalent to a Newtonian fluid, and
3. $n>1$ indicates that the fluid is a dilatant fluid.

The fully developed velocity distribution is derived in terms of mean velocity $(u_m)$ as follows [18]:

In order to illustrate the solution technique , a number of simplifying assumptions are made for the simplified of the basic equation as:

1. the flow mode is laminar, steady and axial symmetry.
2. the fluid physical properties are independent of temperature and pressure,
3. the axial heat conduction is negligible relative to radial heat conduction,
4. natural convection effects are neglected

$$\frac{u(r)}{u_m}=\frac{3n+1}{n+1}\left[1-\left(\frac{r}{R}\right)^{\frac{n+1}{n}}\right] \tag{4}$$

where $(u_m)$ is the mean velocity.

By coupling Eqn. (3) with Eqn. (4), the shear stress for non-Newtonian power-law can be expressed by:

$$t_{rz}=m\left[\left(\frac{u_m}{R}\right)\left(\frac{3n+1}{n}\right)\left(\frac{r}{R}\right)^{1/n}\right]^n \tag{5}$$

In this case, the steady-state heat balance taking viscous dissipation into account is expressed as follows [18]:

$$k\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)=rc_p u(r)\left(\frac{\partial T}{\partial z}\right)-t_{rz}\left(\frac{du}{dr}\right) \tag{6}$$

Where ρ, c and k are the density, specific heat and thermal conductivity, respectively. In addition, the second term on the right-hand side is the viscous dissipationterm effects.

In order to avoid difficulties of definitions the heat transfer and to simplify the mathematical treatments, three modified boundary conditions are proposed and employed for special process requirements as:

-At pipe center; by applying constant centerline temperature gives:

-By symmetry, there can be no heat flux across the centerline in the pipe; this case means that the radial temperature gradient is zero, therefore:

B.C 1 r=0 at center line r=R at wall (7)

$$\text{B.C 2} \qquad \text{at } r=0 \qquad \frac{\partial T}{\partial r}=0 \tag{8}$$

-At pipe wall; the identified temperature gives the third boundary condition:

$$\text{B.C 3} \qquad \text{at } r=R \qquad T=T_w \tag{9}$$

Now, substituting the velocity profile, Eqn. (4), into energy balance Eqn. (6) yields;

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)=\frac{u_m}{a}\left(\frac{3n+1}{n+1}\right)\left[1-\left(\frac{r}{R}\right)^{\frac{n+1}{n}}\right]\left(\frac{\partial T}{\partial z}\right)+\frac{m}{k}\left(\frac{3n+1}{n}\right)^{n+1}\left(\frac{u_m}{R}\right)^{n+1}\left(\frac{r}{R}\right)^{\frac{n+1}{n}} \tag{10}$$

Where $a$ is thermal diffusivity and defined by:

$$a=\frac{k}{rc_p} \tag{11}$$

Generally, one tries to select dimensionless quantities so as to minimize the number of parameter in the final problem formulation and that is useful in scale-up problems by introducing the following dimensionless variables:

$$\left.\begin{aligned} x&=\frac{r}{R};h=\frac{z}{2RPe}\\ q&=\left(\frac{T_w-T(r)}{T_w-T_c}\right)\end{aligned}\right\} \tag{12}$$

Substitution of Eqn. (12) into Eq. (10) yields a dimensionless partial differential equation as:

$$\frac{1}{x}\frac{\partial}{\partial x}\left(x\frac{\partial q}{\partial x}\right)=\frac{1}{4}\left(\frac{3n+1}{n+1}\right)\left[1-x^{\frac{n+1}{n}}\right]\left(\frac{\partial q}{\partial h}\right)-Br\left(\frac{3n+1}{n}\right)^{n+1}x^{\frac{n+1}{n}} \tag{13}$$

Where the Brinkman number $(Br)$ is defined by:

$$Br = -\frac{mu^{n+1}}{kR^{n-1}(T_w - T_c)} \tag{14}$$

and the Peclet number $(Pe)$ is given by :

$$Pe = \frac{2u_m R}{a} \tag{15}$$

Based on these new parameters and Eqns. (7), (8) and (9), the dimensionless boundary conditions becomes as:

B.C: 1 at $x = 0$ ; $q = 1$ (16)

B.C: 2 at $x = 0$ ; $\frac{dq}{dx} = 0$ (17)

B.C: 3 at $x = 1$ ; $q = 0$ (18)

Generally for the case of constant wall temp., radial temperature profiles are well stabilized; so that $q(x,h)$ is a function of dimensional cylindrical coordinate $(x)$ alone, the constancy of the flux implies that:

$$\frac{\partial q}{\partial h} = A_0 \tag{19}$$

where $A_0$ is a constant; substituting Eqn. (19) into Eqn. (13) yields an ordinary differential equation, as:

$$\frac{1}{x}\frac{d}{dx}\left(x\frac{dq}{dx}\right) = \frac{A_0}{4}\left(\frac{3n+1}{n+1}\right)\left[1 - x^{\frac{n+1}{n}}\right] - Br\left(\frac{3n+1}{n}\right)^{n+1} x^{\frac{n+1}{n}} \tag{20}$$

Let introduce new other simplified parameters as:

$$N = \left(\frac{3n+1}{n}\right)^{n+1} Br \tag{21}$$

$$C_0 = \frac{A_0}{4}\left(\frac{3n+1}{n+1}\right) \tag{22}$$

and

$$b = \frac{n+1}{n} \tag{23}$$

Now, substituting Eqns. (21), (22) and (23), into Eqn. (20) yields;

$$\frac{1}{x}\frac{d}{dx}\left(x\frac{dq}{dx}\right) = C_0\left(1-x^{b}\right) - Nx^{b} \tag{24}$$

This separable differential equation can be directly integrated twice with respect to dimensionless radial coordinate $(x)$; the results give temperature profile in circular pipe as follows:

$$q(x) = C_0\left(\frac{1}{4}x^{2} - \frac{1}{(b+2)^2}x^{b+2}\right) - \frac{N}{(b+2)^2}x^{b+2} + C_1 \ln x + C_2 \tag{25}$$

in which $C_1$ and $C_2$ are constants of integration. These two constants can be evaluated from the first two boundary conditions Eqn. (16) and Eqn. (17) and by utilizing of Eqn. (25), as:

$$C_1 = 0 \quad \text{and} \quad C_2 = 1 \tag{26}$$

These two expressions of integration constants can be inserted into Eqn. (25) and rearranged to give the dimensionless temperature profile as follows:

$$q(x) = C_0\left(\frac{1}{4}x^{2} - \frac{1}{(b+2)^2}x^{b+2}\right) - \frac{N}{(b+2)^2}x^{b+2} + 1 \tag{27}$$

Based on the third boundary condition, Eqn. (18), with Eqn. (27), one can find parameter

$$C_0 = 4\left[\frac{\left(N-(b+2)^2\right)}{\left(4-(b+2)^2\right)}\right] \tag{28}$$

When substitution for $C_0$ into Eqn. (27), the local radial temperature distribution becomes as:

$$q(x) = \left(\frac{(N-C_0)}{(b+2)^2}\right)x^{b+2} + \left(\frac{C_0}{4}\right)x^{2} + 1 \tag{29}$$

In another form:

$$\frac{T_w - T(r)}{T_w - T_c} = \left(\frac{(N-C_0)}{\left(\frac{3n+1}{n}\right)^2}\right)\left(\frac{r}{R}\right)^{\frac{3n+1}{n}} + \left(\frac{C_0}{4}\right)\left(\frac{r}{R}\right)^{2} + 1 \tag{30}$$

This equation represents the temperature profile for power-law fluid in cylindrical pipe with viscous effects under the effect of constant wall temp..

In fully-developed flow, it is usual to utilize the bulk temperature (mean fluid temperature), $T_b$ , rather than the center-line temperature when defining the Nusselt number [8,18]. This mean or bulk temperature is given by:

$$T_b = \frac{2p\,rc_p \int_0^{2p}\int_0^R T(r)u(r)rdrdq}{2p\,rc_p \int_0^{2p}\int_0^R u(r)rdrdq} \tag{31}$$

In dimensionless form of bulk temperature $(q_b)$, Eqn. (31) becomes:

$$q_b = \frac{\int_0^1 q(x)f(x)xdx}{\int_0^1 fx(x)dx} \tag{32}$$

Where $f$ is the dimensionless velocity profile and defined as:

$$f = \frac{u}{u_m} \tag{33}$$

By substitution velocity and temperature profiles Eqns. (4) and Eqn. ( 29) into Eqn. (32) becomes:

$$q_b = \frac{\int_0^1 \left\{\left[\left(\frac{(N-C_0)}{(b+2)^2}\right)x^{b+2} + \left(\frac{C_0}{4}\right)x^2 + 1\right]\left(1-x^b\right)\right\}xdx}{\int_0^1 \left(1-x^b\right)xdx} \tag{34}$$

Taking the generality of the analysis into account, this equation, Eqn. (34), can be readily integrated to obtain the dimensionless forms of bulk temperature as follows:

$$q_b == \left[\frac{2(N-C_0)}{(b+2)(b+3)(2b+3)}\right] + \left(\frac{C_0(b+2)}{8(b+4)}\right) + 1 \tag{35}$$

According to the axes that shown in Figure 1, the local heat transfer coefficient inside the pipe is normally defined by:

$$q_w = h(T_w - T_b) = k\left.\frac{dT}{dr}\right|_{r=R} \tag{36}$$

According to the definition the local Nusselt number $(Nu)$ is given by:

$$Nu = \frac{hD}{k} = \frac{2R\left.\frac{dT}{dr}\right|_{r=R}}{(T_w - T_b)} \tag{37}$$

Based on dimensionless parameters, the local Nusselt number $(Nu)$ is can be expressed by:

$$Nu = 2\frac{\left.\frac{dq}{dx}\right|_{x=1}}{(q_w - q_b)} \tag{38}$$

Where $q_W$ is the dimensionless wall temperatures, which could be evaluated by $q_w = q|_{x=1}$, therefore when substitution in Eqn. (29), the results could be as:

$$q_w = q|_{x=1} = \left[\frac{(N - C_0)}{(b+2)^2}\right] + \left(\frac{C_0}{4}\right) + 1 \tag{39}$$

In addition, from Eqn. (29), the temperature gradient at wall $\left(\left.\frac{dq}{dx}\right|_{x=1}\right)$ is given as:

$$\left.\frac{dq}{dx}\right|_{x=1} = \frac{(2N + bC_0)}{2(b+2)} \tag{40}$$

By substitution the Eqns. (35), (39) and (40) into Eqn. (38) and simplified the results, the final results of Nusselt number $(Nu)$ can be evaluated and expressed as:

$$Nu = \left[\frac{31n^2 + 12n + 1}{8(3n+1)(5n+1)} + 2^{n-1} Br\left(\frac{3n+1}{n}\right)^n\right]^{-1} \tag{41}$$

This equation represents the Nusselt number for power-law fluid in cylindrical pipe with viscous effects under the effect of constant wall temp. for fully developed laminar flow.

## 3. Results and discussion

In the absence of dissipation effect (Br=0) the solution is independent of whether there is wall heating or cooling. However, viscous dissipation always contributes to internal heating of the fluid; hence the solution will differ according to the process taking place. The Brinkman number (Br) is chosen as a criterion which shows the relative importance of viscous dissipation. For brevity and standing in a reasonable range, -1 < Br < 1. Where positive values of *Br* correspond to wall heating ($T_w > T_c$ and Br>0) case that mean heat is being supplied across the walls into the fluid, while the opposite is true for negative values of Br, that mean wall cooling cases ($T_w < T_c$ and Br<0).

As stated earlier that the thermal boundary conditions have been considered for the pipe wall as constant wall temp.. For this boundary condition both wall heating or wall cooling cases are examined and treated separately.

Figures (2a, b and c) show the temperature profiles made dimensionless using this scale for wall heating, no viscous dissipation and wall cooling cases, respectively, where these profiles based on Eqn. (29). These plots make clear the aforementioned effects of increased dissipation. As expected, increasing dissipation increases the bulk temperature of the fluid due to internal heating of the fluid. For the wall heating case, this increase in the fluid temperature decreases the temperature difference between the wall and the fluid, as will be shown later, which is followed with a decrease in heat transfer. When wall cooling is applied, due to the internal heating effect of the viscous dissipation on the fluid temperature profile, temperature difference is increased with the increasing Brinkman number $(Br)$. In fact, wall cooling is applied to reduce the bulk temperature of the fluid, while the effect of the viscous dissipation is increasing the bulk temperature of the fluid. Therefore, the amount of viscous dissipation may change the overall heat balance. When the Brinkman number exceeds a certain limiting value, the heat generated internally by viscous dissipation process will overcome the effect of wall cooling.

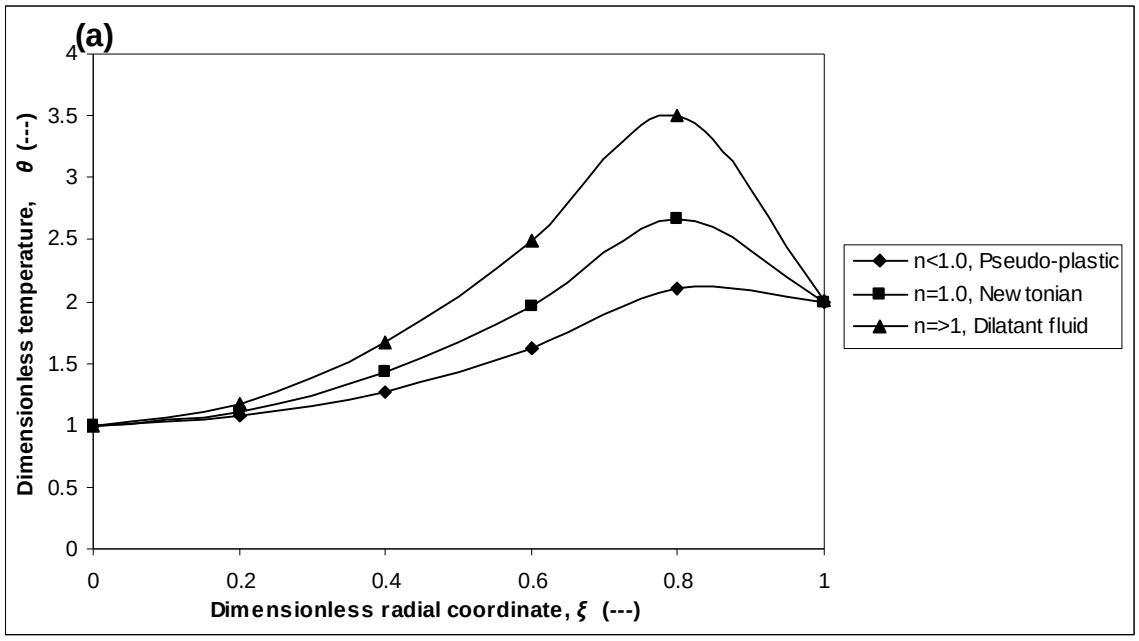


**Figure (2- a)**

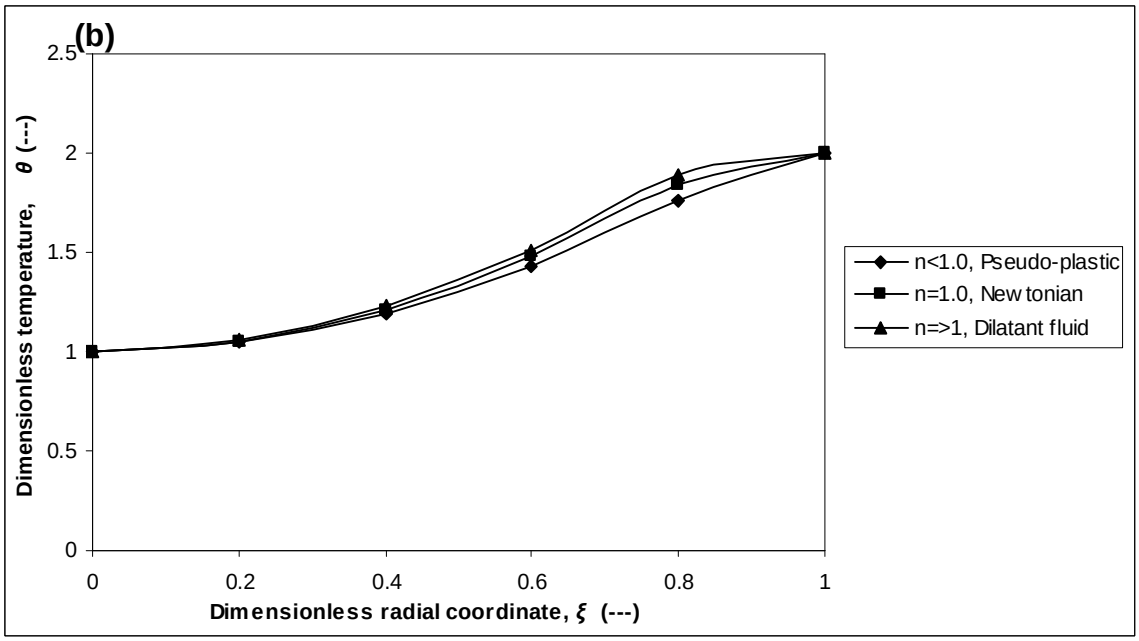


**Figure (2-b)**

**Figure (2). Effects of power law index on dimensionless fluid temperature profiles for: (a) heating wall (Br=-1.0), (b) no viscous dissipation (Br=0.0), and (c) cooling wall (Br=1.0).**

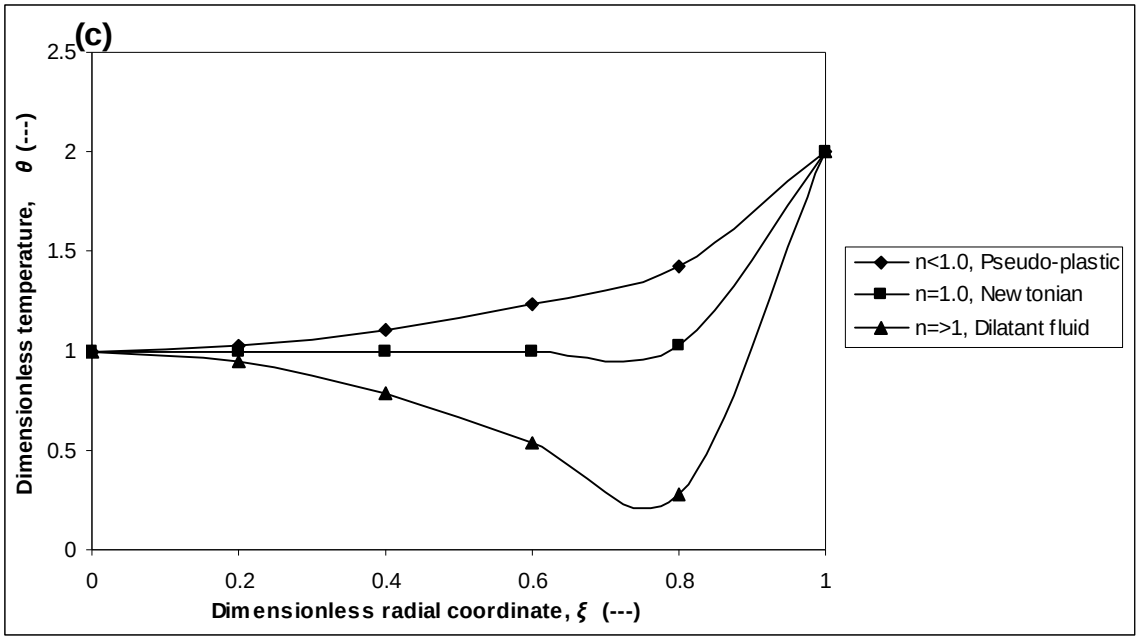


**Figure (2-c)**

Figure (3) represents the variation of Nusselt number with the rheological properties (power-law index) for constant wall temp. case with different values of Brinkman number. Where the asymptotic and downstream Nusselt number profiles are shown clearly for wall heating (Br > 0). When wall cooling (Br < 0) is applied to reduce the bulk temperature of the fluid, as explained earlier, the amount of viscous dissipation may change the overall heat balance. With increasing value of Br in the negative direction, the Nusselt number reaches an asymptotic value. As noticed, when Br goes to infinity for either the wall heating or the wall cooling case, the Nusselt number reaches the same asymptotic value. This is due to the fact that the heat generated internally by viscous dissipation processes will balance the effect of wall cooling. Generally, Nusselt number with viscous effects for both wall heating and wall cooling is less than Nusselt number for non viscous dissipation.

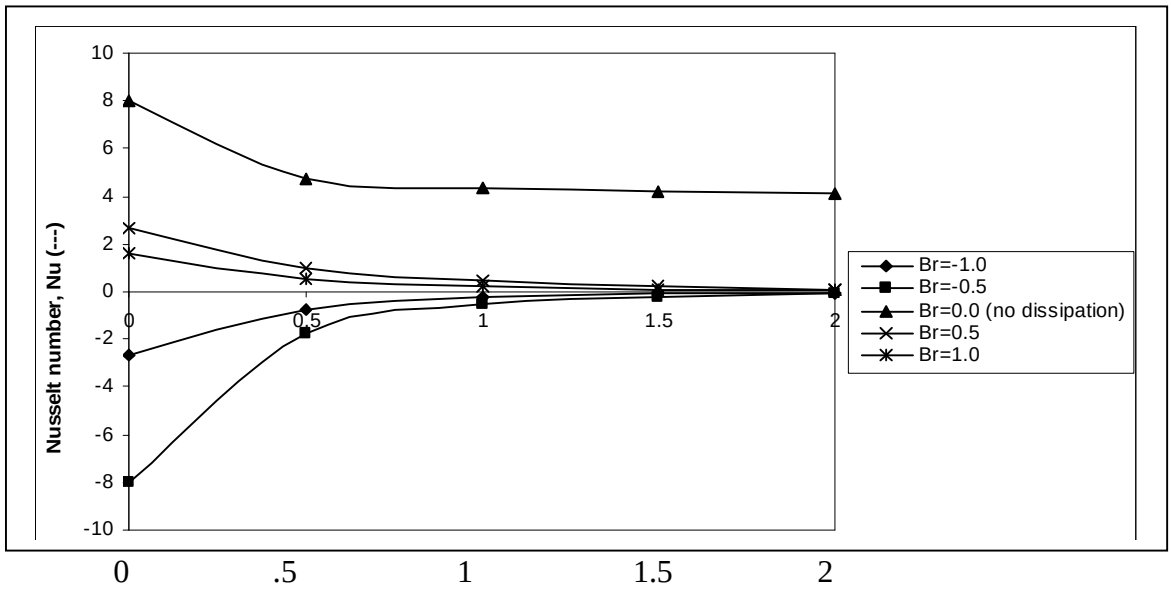


**Figure (3). Effects of power-law index on Nusselt number for different values of Brinkman number.**

While Figure (4) represents the variation of Nusselt number with the Brinkman number for constant wall temp. case with different values of power law index (n). Actually, this is an expected result, when Eqn. (41) is closely examined. For the wall heating case, with the increasing value of Br, Nu decreases to reach constant values. This is because the temperature difference which drives the heat transfer decreases. At Br=0.5, the heat supplied by the wall into the fluid is balanced with the internal heat generation due to the viscous heating. For Br > 0.5, the internally generated heat by the viscous dissipation overcomes the wall heat. When Br=1.0, Nu reaches an asymptotic value. Generally, Nusselt number Newtonian fluid is higher than those for pseudo- plastic and dilatants fluids.

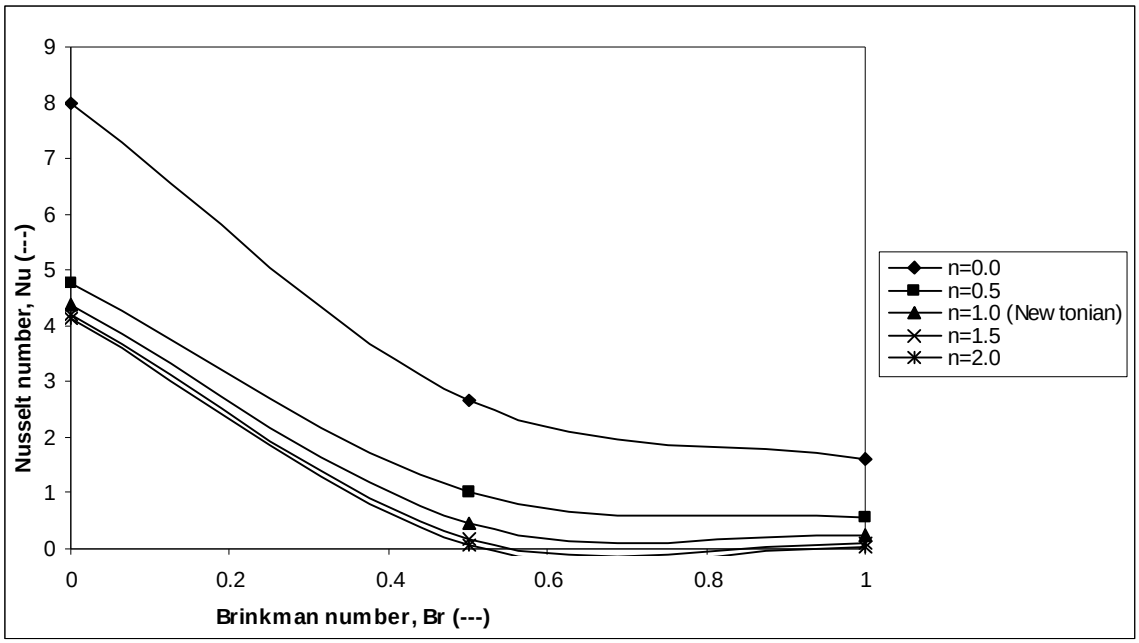


**Figure (4). Effects of Brinkman number on Nusselt number for different values of power-law index.**

## 4. Conclusions

The forced convection heat transfer problem with viscous dissipation inside a pipe subjected to constant wall temp. has been solved mathematically, which is a type of the Graetz problem. Completely analytical solutions for the fluid temperature and localNusselt number (Nu) have been derived. The effects of the Brinkman number (Br) and rheological properties (power-law index) on the distribution of the local Nusselt number have been shown through numerical calculations. The local Nusselt number in the thermal region tends to increase with a decrease in the power-law model index (n). It has been shown that viscous dissipation in the fluid can significantly influence laminar flow heat transfer.

With regard to the Graetz problem, the present analytical method can be applied to heat transfer not only in a pipe but also in a concentric annulus and a channel between parallel plates; it can also be applied to heat transfer in a channel with a moving wall because there is no restriction on the velocity distribution form of a fluid.

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## 6. Nomenclature

**List of symbols**

| | |
|---|---|
| Br | Brinkman number = $mu_m^{n+1}R^{n-1}/ [k (T_w - T_c)]$ |
| $c_p$ | specific heat (J $kg^{-1}$ $K^{-1}$) |
| h | local heat transfer coefficient ($W/m^2$ .K) |
| n | power-law model parameter ($Pa.s^n$ ) |
| Nu | local Nusselt number = 2hR/k |
| r | radial coordinate (m). |
| R | pipe radius (m) |
| T | fluid temperature (K) |
| u | axial fluid velocity (m/s) |
| $u_m$ | mean axial fluid velocity (m/s) |

| | |
|---|---|
| Pe | Peclet number = $2u_mR/a$ |
| $q_w$ | constant heat flux ($W/m^2$) |
| z | axial coordinate (m). |

**Greek symbols**

| | |
|---|---|
| η | dimensionless axial coordinate $=z/(u_mR^2 . a)$ |
| k | thermal conductivity (W/m K) |
| θ | dimensionless fluid temperature = $(T_w - T)/(T_w - T_c)$ |
| $f$ | dimensionless velocity = $u/u_m$ |
| $a$ | thermal diffusivity ($m^2/s$) |
| ρ | fluid density ($kgm^{-3}$) |
| $\tau_{rx}$ | shear stress ($Pa/m^2$) |
| ξ | dimensionless radial coordinate = r/R |

**Subscripts**

| | |
|---|---|
| b | bulk |
| c | centerline |
| w | wall |